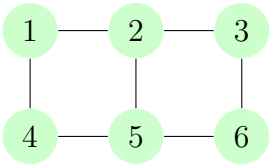
MATH 579: Combinatorics Exam 7

Please read the following instructions. For the following exam you may not use any papers, books, or computers. You may use a calculator. Please turn in **exactly four** problems. You must do problems 1-3, and one more chosen from 4-6. Please write your answers on separate paper, make clear what work goes with which problem, adequately justify all answers, simplify all numerical answers as best you can, and put your name or initials on every page. You have 50 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by $\frac{5}{2}$ for your exam score.

Turn in problems 1,2,3:

1. Find the automorphism group of the following graph. Give all elements explicitly, in standard (cycle) form.

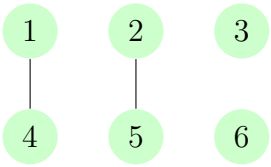


2. In S_4, set $\pi = (1,2,3), \tau = (2,3,4)$. Find all elements in the subgroup $\langle \pi, \tau \rangle$. Give all elements explicitly, in standard (cycle) form.
HINT: There are fewer than 24 elements.

3. Use Burnside's Lemma to compute the number of ways to color the vertices of K_5, distinct up to automorphism, drawn from 3 possible colors.

Turn in exactly one more problem of your choice:

4. Use Burnside's Lemma to compute the number of ways to color the vertices of the following graph, distinct up to automorphism, drawn from 4 possible colors.



5. Use Burnside's Lemma to compute the number of ways to color the vertices of C_{12}, distinct up to automorphism, drawn from 2 possible colors.

6. Let p be an odd prime. Use Burnside's Lemma to compute the number of ways to color the vertices of C_p, distinct up to automorphism, drawn from n possible colors.

Please keep this sheet for your records.